EXAMPLE 8 Solve: $\log_7(x+1) + \log_7(x-1) = \log_7 8$.

ALGEBRAIC APPROACH

We have

$$\log_7(x+1) + \log_7(x-1) = \log_7 8$$

$$\log_7[(x+1)(x-1)] = \log_7 8 \quad \text{Using the product rule for logarithms}$$

$$\log_7(x^2-1) = \log_7 8 \quad \text{Multiplying. Note that both sides are base-7 logarithms.}$$

$$x^2 - 1 = 8 \quad \text{Using the principle of logarithmic equality. Study this step carefully.}$$

$$x^2 - 9 = 0$$

$$(x-3)(x+3) = 0 \quad \text{Solving the quadratic equation}$$

$$x = 3 \quad \text{or} \quad x = -3.$$

The student should confirm that 3 checks but −3 does not. The solution is 3.

GRAPHICAL APPROACH

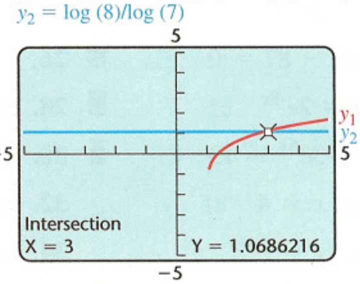
Using the change-of-base formula, we graph

$$y_1 = \log(x+1)/\log(7) + \log(x-1)/\log(7)$$

and

$$y_2 = \log(8)/\log(7).$$

$y_1 = \log(x+1)/\log(7) + \log(x-1)/\log(7)$,
$y_2 = \log(8)/\log(7)$



The graphs intersect at (3, 1.0686216). The solution is 3.

Try Exercise 57.

9.6 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *In each of Exercises 1–8, match the equation with an equivalent equation from the column on the right that could be the next step in the solution process.*

1. (e) $5^x = 3$
2. (a) $e^{5x} = 3$
3. (f) $\ln x = 3$
4. (h) $\log_x 5 = 3$
5. (b) $\log_5 x + \log_5(x-2) = 3$
6. (d) $\log_5 x - \log_5(x-2) = 3$
7. (g) $\ln x - \ln(x-2) = 3$
8. (c) $\log x + \log(x-2) = 3$

a) $\ln e^{5x} = \ln 3$

b) $\log_5(x^2 - 2x) = 3$

c) $\log(x^2 - 2x) = 3$

d) $\log_5 \dfrac{x}{x-2} = 3$

e) $\log 5^x = \log 3$

f) $e^3 = x$

g) $\ln \dfrac{x}{x-2} = 3$

h) $x^3 = 5$

Solve. Where appropriate, include approximations to three decimal places.

9. $3^{2x} = 81$ 2

10. $2^{3x} = 64$ 2

11. $4^x = 32$ $\frac{5}{2}$

12. $9^x = 27$ $\frac{3}{2}$

13. $2^x = 10$ □

14. $2^x = 24$ □

15. $2^{x+5} = 16$ -1

16. $2^{x-1} = 8$ 4

17. $8^{x-3} = 19$ □

18. $5^{x+2} = 15$ □

19. $e^t = 50$ □

20. $e^t = 20$ □

21. $e^{-0.02t} = 8$ □

22. $e^{-0.01t} = 100$ □

23. $5 = 3^{x+1}$ □

24. $7 = 3^{x-1}$ □

25. $4.9^x - 87 = 0$ □

26. $7.2^x - 65 = 0$ □

27. $19 = 2e^{4x}$ □

28. $29 = 3e^{2x}$ □

29. $7 + 3e^{5x} = 13$ □

30. $4 + 5e^{4x} = 9$ 0

Aha! **31.** $\log_3 x = 4$ 81

32. $\log_2 x = 6$ 64

33. $\log_2 x = -3$ $\frac{1}{8}$

34. $\log_5 x = 3$ 125

35. $\ln x = 5$ $e^5 \approx 148.413$

36. $\ln x = 4$ $e^4 \approx 54.598$

37. $\ln (4x) = 3$ □

38. $\ln (3x) = 2$ □

39. $\log x = 2.5$ □

40. $\log x = 0.5$ □

41. $\ln (2x + 1) = 4$ □

42. $\ln (4x - 2) = 3$ □

Aha! **43.** $\ln x = 1$ $e \approx 2.718$

44. $\log x = 1$ 10

45. $5 \ln x = -15$ $e^{-3} \approx 0.050$

46. $3 \ln x = -3$ $e^{-1} \approx 0.368$

47. $\log_2 (8 - 6x) = 5$ -4

48. $\log_5 (2x - 7) = 3$ 66

49. $\log (x - 9) + \log x = 1$ 10

50. $\log (x + 9) + \log x = 1$ 1

51. $\log x - \log (x + 3) = 1$ No solution

52. $\log x - \log (x + 7) = -1$ $\frac{7}{9}$

Aha! **53.** $\log (2x + 1) = \log 5$ 2

54. $\log (x + 1) - \log x = 0$ No solution

55. $\log_4 (x + 3) = 2 + \log_4 (x - 5)$ $\frac{83}{15}$

56. $\log_2 (x + 3) = 4 + \log_2 (x - 3)$ $\frac{17}{5}$

57. $\log_7 (x + 1) + \log_7 (x + 2) = \log_7 6$ 1

58. $\log_6 (x + 3) + \log_6 (x + 2) = \log_6 20$ 2

59. $\log_5 (x + 4) + \log_5 (x - 4) = \log_5 20$ 6

60. $\log_4 (x + 2) + \log_4 (x - 7) = \log_4 10$ 8

61. $\ln (x + 5) + \ln (x + 1) = \ln 12$ 1

62. $\ln (x - 6) + \ln (x + 3) = \ln 22$ 8

63. $\log_2 (x - 3) + \log_2 (x + 3) = 4$ 5

64. $\log_3 (x - 4) + \log_3 (x + 4) = 2$ 5

□ Answers to Exercises 13, 14, 17–29, and 37–42 are on p. IA-26.

65. $\log_{12} (x + 5) - \log_{12} (x - 4) = \log_{12} 3$ $\frac{17}{2}$

66. $\log_6 (x + 7) - \log_6 (x - 2) = \log_6 5$ $\frac{17}{4}$

67. $\log_2 (x - 2) + \log_2 x = 3$ 4

68. $\log_4 (x + 6) - \log_4 x = 2$ $\frac{2}{5}$

69. $e^{0.5x} - 7 = 2x + 6$ $-6.480, 6.519$

70. $e^{-x} - 3 = x^2$ -1.873

71. $\ln (3x) = 3x - 8$ $0.000112, 3.445$

72. $\ln (x^2) = -x^2$ $-0.753, 0.753$

73. Find the value of x for which the natural logarithm is the same as the common logarithm. 1

74. Find all values of x for which the common logarithm of the square of x is the same as the square of the common logarithm of x. 1, 100

TW **75.** Christina finds that the solution of $\log_3 (x + 4) = 1$ is -1, but rejects -1 as an answer because it is negative. What mistake is she making?

TW **76.** Could Example 2 have been solved by taking the natural logarithm on both sides? Why or why not?

SKILL REVIEW

To prepare for Section 9.7, review using the five-step problem-solving strategy.

Solve.

77. A rectangle is 6 ft longer than it is wide. Its perimeter is 26 ft. Find the length and the width. [1.7]
Length: 9.5 ft; width: 3.5 ft

78. Under one health insurance plan offered in California, the maximum co-pay for an individual is \$3000 per calendar year. The co-pay for each visit to a specialist is \$40, and the co-pay for a hospitalization is \$1000. With hospitalizations and specialist visits, Marguerite reached the maximum co-pay in 2010. If she was hospitalized twice, how many visits to specialists did she make? [4.1] 25 visits or more
Source: ehealthinsurance.com

79. Joanna wants to mix Golden Days bird seed containing 25% sunflower seeds with Snowy Friends bird seed containing 40% sunflower seeds. She wants 50 lb of a mixture containing 33% sunflower seeds. How much of each type should she use? [3.3]
Golden Days: $23\frac{1}{3}$ lb; Snowy Friends: $26\frac{2}{3}$ lb

80. The outside edge of a picture frame measures 12 cm by 19 cm, and 144 cm^2 of picture shows. Find the width of the frame. [5.8] 1.5 cm

81. Max can key in a musical score in 2 hr. Miles takes 3 hr to key in the same score. How long would it take them, working together, to key in the score? [6.5] $1\frac{1}{5}$ hr

82. A sign is in the shape of a right triangle. The hypotenuse is 3 ft long, and the base and the height of the triangle are equal. Find the length of the base and the height. Round to the nearest tenth of a foot. [9.7] Approximately 2.1 ft

SYNTHESIS

TW **83.** Can the principle of logarithmic equality be expanded to include all functions? That is, is the statement "$m = n$ is equivalent to $f(m) = f(n)$" true for any function f? Why or why not?

TW **84.** Explain how Exercises 39 and 40 could be solved using the graph of $f(x) = \log x$.

Solve. If no solution exists, state this.

85. $27^x = 81^{2x-3}$ $\frac{12}{5}$

86. $8^x = 16^{3x+9}$ -4

87. $\log_x (\log_3 27) = 3$ $\sqrt[3]{3}$

88. $\log_6 (\log_2 x) = 0$ 2

89. $x \log \frac{1}{8} = \log 8$ -1

90. $\log_5 \sqrt{x^2 - 9} = 1$ $\pm\sqrt{34}$

91. $2^{x^2+4x} = \frac{1}{8}$ $-3, -1$

92. $\log (\log x) = 5$ $10^{100,000}$

93. $\log_5 |x| = 4$ $-625, 625$

94. $\log x^2 = (\log x)^2$ 1, 100

95. $\log \sqrt{2x} = \sqrt{\log 2x}$ $\frac{1}{2}, 5000$

96. $1000^{2x+1} = 100^{3x}$ No solution

97. $3^{x^2} \cdot 3^{4x} = \frac{1}{27}$ $-3, -1$

98. $3^{3x} \cdot 3^{x^2} = 81$ $-4, 1$

99. $\log x^{\log x} = 25$ $\frac{1}{100,000}, 100,000$

100. $3^{2x} - 8 \cdot 3^x + 15 = 0$ $1, \dfrac{\log 5}{\log 3} \approx 1.465$

101. $(81^{x-2})(27^{x+1}) = 9^{2x-3}$ $-\frac{1}{3}$

102. $3^{2x} - 3^{2x-1} = 18$ $\frac{3}{2}$

103. Given that $2^y = 16^{x-3}$ and $3^{y+2} = 27^x$, find the value of $x + y$. 38

104. If $x = (\log_{125} 5)^{\log_5 125}$, what is the value of $\log_3 x$? -3

Try Exercise Answers: Section 9.6

9. 2 **17.** $\dfrac{\log 19}{\log 8} + 3 \approx 4.416$ **21.** $\dfrac{\ln 8}{-0.02} \approx -103.972$
47. -4 **49.** 10 **57.** 1 **65.** $\frac{17}{2}$ **69.** $-6.480, 6.519$

9.7 Applications of Exponential and Logarithmic Functions

- Applications of Logarithmic Functions
- Applications of Exponential Functions

We now consider applications of exponential and logarithmic functions.

APPLICATIONS OF LOGARITHMIC FUNCTIONS

EXAMPLE 1 Sound Levels. To measure the volume, or "loudness," of a sound, the *decibel* scale is used. The loudness L, in decibels (dB), of a sound is given by

$$L = 10 \cdot \log \frac{I}{I_0},$$

where I is the intensity of the sound, in watts per square meter (W/m^2), and $I_0 = 10^{-12}\ \text{W/m}^2$. ($I_0$ is approximately the intensity of the softest sound that can be heard by the human ear.)

a) The average maximum intensity of sound in a New York subway car is about $3.2 \times 10^{-3}\ \text{W/m}^2$. How loud, in decibels, is the sound level?
Source: Columbia University Mailman School of Public Health

b) The Occupational Safety and Health Administration (OSHA) considers sustained sound levels of 90 dB and above unsafe. What is the intensity of such sounds?